= ИНТЕГРАЛИ =

1)

$$\int \frac{1+\sqrt{x-1}}{1-\sqrt{x-1}}\,dx = \int \frac{1+(1-t)}{t}(2t-2)\,dt = \int \frac{(2-t)2(t-1)}{t}\,dt =$$

$$= 2\cdot\int \frac{2t-2-t^2+t}{t}\,dt = 2\int \frac{-t^2+3t-2}{t}\,dt$$

$$= 2\cdot\int\left(-t+3-\frac{2}{t}\right)dt = 2\left(-\frac{t^2}{2}+3t-2\ln t\right)+C$$

$$= -\left(1-\sqrt{x-1}\right)^2+6\left(1-\sqrt{x-1}\right)-2\ln\left(1-\sqrt{x-1}\right)+C$$

смена

$1-\sqrt{x-1}=t \Rightarrow$

$1-t=\sqrt{x-1}\ /()^2$

$1-2t+t^2=x-1$

$2-2t+t^2=x\ /'$

$(-2+2t)\,dt=dx$

2)

$$\int \frac{2x-3}{3\sqrt{x}}\,dx = \frac{1}{3}\int\frac{2x-3}{x^{\frac{1}{2}}}\,dx = \frac{1}{3}\int\left(2x^{1-\frac{1}{2}}-3x^{-\frac{1}{2}}\right)dx = \frac{1}{3}\int\left(2x^{\frac{1}{2}}-3x^{-\frac{1}{2}}\right)dx =$$

$$= \frac{2}{3}\cdot\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C = \frac{2}{3}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \frac{4}{9}\sqrt{x^3}-2\sqrt{x}+C$$

3)

$$\int \overset{u}{x^2}\cdot \overset{dv}{e^{-\frac{x}{2}}}\,dx = \left[u=x^2 \Rightarrow du=2x\,dx;\ dv=e^{-\frac{x}{2}}dx \Rightarrow v=\int e^{-\frac{x}{2}}dx = \frac{1}{-\frac{1}{2}}e^{-\frac{x}{2}} = -2e^{-\frac{x}{2}}\right]$$

(со извод) (со интеграл)

$$= -2x^2e^{-\frac{x}{2}} - \int -2e^{-\frac{x}{2}}\cdot 2x\,dx = -2x^2e^{-\frac{x}{2}} + 4\int \underset{u}{x}\cdot \underset{dv}{e^{-\frac{x}{2}}}\,dx =$$

(на страна) I_1

$$= -2x^2e^{-\frac{x}{2}} + 4\cdot\left[-2xe^{-\frac{x}{2}} - 4e^{-\frac{x}{2}}\right]+C =$$

$$= e^{-\frac{x}{2}}\cdot\left(-2x^2-8x-4\right)+C$$

Парцијална интеграција

$$\int u\cdot dv = u\cdot v - \int v\cdot du$$

таблично

$$\int e^{ax}dx = \frac{1}{a}e^{ax}+C$$

доказ!

$ax=t\ /'$

$a\,dx=dt$

$dx=\frac{1}{a}dt$

$$\int e^{ax}dx = \frac{1}{a}\int e^t dt = \frac{1}{a}e^t+C = \frac{1}{a}e^{ax}+C$$

$$I_1 = \int \overset{u}{x}\,\overset{dv}{e^{-\frac{x}{2}}}\,dx = -2xe^{-\frac{x}{2}} - \int -2e^{-\frac{x}{2}}dx = -2xe^{-\frac{x}{2}} + 2\int e^{-\frac{x}{2}}dx =$$

$u=x \Rightarrow du=dx$

$dv=e^{-\frac{x}{2}}dx \Rightarrow v=\int e^{-\frac{x}{2}}dx = -2e^{-\frac{x}{2}}$

$$= -2xe^{-\frac{x}{2}} + 2\cdot\left(-2e^{-\frac{x}{2}}\right)+C = -2xe^{-\frac{x}{2}} - 4e^{-\frac{x}{2}}+C$$

5)

$$I = \int \frac{x\cdot\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{1+x^2}}\,dx = \left[\begin{array}{l} u=x \\ du=dx \end{array}\right.$$

лоша идеја

$$dv = \frac{\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{1+x^2}}\,dx$$

$$v = \int \frac{\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{1+x^2}}\,dx = ?$$

($\frac{dx}{\sqrt{1+x^2}}$ → dt, со смена)

$t = \ln(x+\sqrt{x^2+1})$

$dt = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(\frac{1}{1} + \frac{1\cdot 2x}{2\sqrt{x^2+1}}\right)dx = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{\frac{x+\sqrt{x^2+1}}{1}}dx = \frac{dx}{\sqrt{x^2+1}} = dt$

$v = \int t\cdot dt = \frac{t^2}{2} = \frac{\ln^2(x+\sqrt{x^2+1})}{2}$

$I = \frac{x}{2}\ln^2(x+\sqrt{x^2+1}) - \frac{1}{2}\int \ln^2(x+\sqrt{x^2+1})dx$

се усложнува (затоа идејата е лоша)

Нова идеја! (подобра)

$u = \ln(x+\sqrt{x^2+1}) \Rightarrow du = \frac{dx}{\sqrt{x^2+1}}$ (решен на почеток на лист)

$dv = \frac{x\,dx}{\sqrt{1+x^2}} \Rightarrow v = \int \frac{x\,dx}{\sqrt{1+x^2}} = \left[t = \sqrt{1+x^2} \Rightarrow t^2 = 1+x^2 \Rightarrow 2t\,dt = 2x\,dx\right] =$

$= \int \frac{t\,dt}{t} = \int dt = t = \sqrt{1+x^2}$

$I = \sqrt{1+x^2}\cdot\ln(x+\sqrt{x^2+1}) - \int \sqrt{1+x^2}\cdot\frac{dx}{\sqrt{x^2+1}} = \sqrt{1+x^2}\cdot\ln(x+\sqrt{x^2+1}) - x + C$

(6) $\int \frac{\ln 3x}{\sqrt[3]{x^2}}dx = \int x^{-\frac{2}{3}}\cdot \ln 3x\cdot dx = 3\sqrt[3]{x}\ln 3x - \int 3x^{\frac{1}{3}}\cdot\frac{1}{x}dx = (*)$

$\left[\begin{array}{l} u = \ln 3x \Rightarrow du = \frac{1}{3x}\cdot 3dx = \frac{1}{x}dx \\ dv = x^{-\frac{2}{3}}dx \Rightarrow v = \int x^{-\frac{2}{3}}dx = \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} = \frac{x^{\frac{1}{3}}}{\frac{1}{3}} = 3x^{\frac{1}{3}} \end{array}\right]$

$(*) = 3\sqrt[3]{x}\ln 3x - 3\cdot\int x^{\frac{1}{3}-1}dx = 3\sqrt[3]{x}\ln 3x - 3\int x^{-\frac{2}{3}}dx = 3\sqrt[3]{x}\ln 3x - 3\cdot(3x^{\frac{1}{3}}) + C =$

$= 3\sqrt[3]{x}\cdot(\ln 3x - 3) + C$

(7) $\int \frac{e^{\sqrt{x}}}{2}dx = \frac{1}{2}\cdot\int e^t\cdot 2t\,dt = \int t\cdot e^t dt = te^t - \int e^t dt = te^t - e^t + C = e^t(t-1) + C =$

смена $\left[\begin{array}{l} t = \sqrt{x} \\ t^2 = x \\ 2t\,dt = dx \end{array}\right]$ $\left[\begin{array}{l} u = t \Rightarrow du = dt \\ dv = e^t dt \Rightarrow v = \int e^t dt = e^t \end{array}\right]$

$= e^{\sqrt{x}}\cdot(\sqrt{x}-1) + C$

(8) $\int \sqrt[3]{x}\cdot\ln^2 x\,dx = \int \sqrt[3]{e^t}\cdot t^2\cdot e^t dt = \int e^{\frac{t}{3}}\cdot t^2\cdot e^t dt = \int t^2\cdot e^{\frac{4}{3}t}dt = (*)$ (задачата се сведува на (3) задача)

$\ln x = t$, $x = e^t$, $dx = e^t dt$ $\left[\begin{array}{l} u = t^2 \Rightarrow du = 2t\,dt \\ dv = e^{\frac{4}{3}t}dt \Rightarrow v = \int e^{\frac{4}{3}t}dt = \frac{1}{\frac{4}{3}}e^{\frac{4}{3}t} = \frac{3}{4}e^{\frac{4}{3}t} \end{array}\right]$ $(*) = \frac{3}{4}t^2e^{\frac{4}{3}t} - \int \frac{3}{4}e^{\frac{4}{3}t}\cdot 2t\,dt =$

$= \frac{3}{4}\ln^2 x\cdot x^{\frac{4}{3}} - \frac{3}{2}\int t\cdot e^{\frac{4}{3}t}dt = \ldots\ldots = \frac{3}{4}\sqrt[3]{x^4}\cdot\ln^2 x - \frac{3}{2}\int t\cdot e^{\frac{4}{3}t}dt = \ldots\ldots$