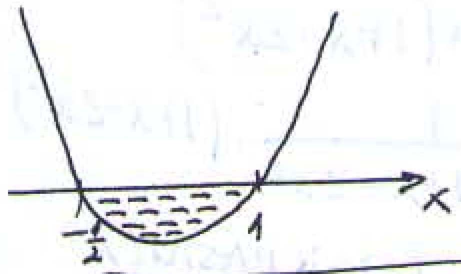
$y = \ln(1+x-2x^2)$

1° D_f. $1+x-2x^2 > 0 \;/\cdot(-1)$

$2x^2 - x - 1 < 0 \longrightarrow$



$x_{1/2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1\pm 3}{4}$

$x_1 = 1 \qquad x_2 = -\frac{1}{2}$

$\boxed{D_f:\ x \in \left(-\frac{1}{2}, 1\right)}$

2° ~~Pošto f-ja nije definisana za negativne x, možemo reći da~~ ne važi ni $f(x) = f(-x)$, ni f-ja nije parna, nije neparna, t.j.

$f(-x) = \ln(1+(-x)-2(-x)^2) = \ln(1-x-2x^2) \neq f(x) \Rightarrow f(-x) = -f(x)$

$\neq -f(x)$

NI PARNA, NI NEPARNA

3° F-ja nije periodična jer $\omega = 0$, t.j. $f(x+\omega) = f(x)$ samo za $\omega = 0$

4° Nule ; $y = 0$ $\quad \ln(1+x-2x^2) = 0$

$1+x-2x^2 = e^0$

$1+x-2x^2 = 1$

$x - 2x^2 = 0$

$x(1-2x) = 0$

$x_1 = 0 \ \vee\ 1-2x = 0$

$2x = 1$

$x_2 = \frac{1}{2}$

$\boxed{T_1(0,0) \quad T_2\left(\frac{1}{2}, 0\right)}$

presek sa x-osom

5° Presek sa y-osom

$x = 0$

$y = \ln(1+0-2\cdot 0^2)$

$y = \ln 1 \qquad \boxed{O(0,0)}$

$y = 0$

6° Znak f-je $\ln(1+x-2x^2) > 0 \Rightarrow 1+x-2x^2 > \cancel{e^0}\ 1$

$x - 2x^2 > 0 \rightarrow$

$x_1 = 0 \quad x_2 = \frac{1}{2}$

$\boxed{x \in \left(0, \frac{1}{2}\right) \text{ f-ja je pozitivna (iznad x-ose)}}$

$\ln(1+x-2x^2) < 0$

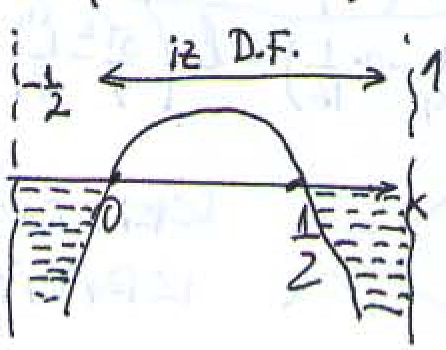
$1+x-2x^2 < e^0$

$1+x-2x^2 < 1$

$x - 2x^2 < 0$

$x(1-2x) < 0$

$x_1 = 0 \quad x_2 = \frac{1}{2}$



$\boxed{x \in \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, 1\right) \text{ f-ja je negativna (ispod x-ose)}}$

7° V.A.

$x = a \Leftrightarrow \lim\limits_{x \to a^{\pm}} f(x) = \pm\infty$

Obično za V.A. gledamo u D_f.

$\lim\limits_{x \to -\frac{1}{2}} \ln(1+x-2x^2) = \ln 0 = -\infty$

$\lim\limits_{x \to 1} \ln(1+x-2x^2) = \ln 0 = -\infty$

$\boxed{x = -\frac{1}{2} \quad x = 1}$ su dve vertikalne asimptote

H.A.

$y = b \Leftrightarrow b = \lim\limits_{x \to \pm\infty} f(x)$

H.A. NEMA JER TREBA $x \to \pm\infty$, a to nije u D_f.

K.A.

$y = ax + b$

$a = \lim\limits_{x \to \pm\infty} \frac{f(x)}{x}$

$b = \lim\limits_{x \to \pm\infty} [f(x) - ax]$

P.S. AKO IMA H.A. ONDA NEMA KOSA ASIMPTOTA (TEOREM). OBRATNO NE VAŽI.

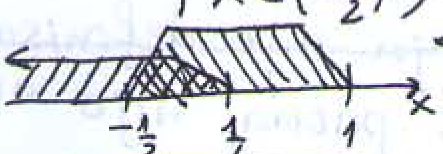
8° $y=\ln(1+x-2x^2)$ $\boxed{D_f=\left(-\frac{1}{2},1\right)}$

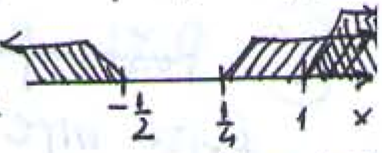
$$y'=\frac{1}{1+x-2x^2}\cdot(1+x-2x^2)'=\frac{1}{1+x-2x^2}\cdot(1-4x)=\frac{1-4x}{1+x-2x^2}$$

$y'>0$ f-ja je RASTUĆA

$$\frac{1-4x}{1+x-2x^2}>0 \Rightarrow \begin{cases}1-4x>0\\ 1+x-2x^2>0\end{cases} \vee \begin{cases}1-4x<0\\ 1+x-2x^2<0\end{cases}$$

$$\begin{cases}x<\frac{1}{4}\\ x\in\left(-\frac{1}{2},1\right)\end{cases} \vee \begin{cases}x>\frac{1}{4}\\ x\in\left(-\infty,-\frac{1}{2}\right)\cup(1,\infty)\end{cases}$$

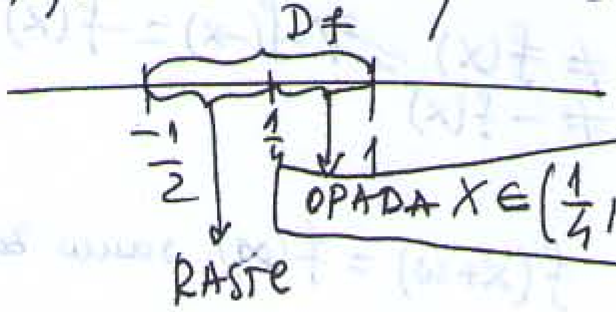




Ako $\boxed{x\in\left(-\frac{1}{2},\frac{1}{4}\right)}$ ONDA f-ja je RASTUĆA

~~$x\in(1,\infty)$~~ OVO NE PRIPADA u D_f.

U ostalim intervalima D_f f-ja će biti opadajuća



$\boxed{\text{OPADA } x\in\left(\frac{1}{4},1\right)}$

9° Ekstremi $y'=0$ $\frac{1-4x}{1+x-2x^2}=0 \Rightarrow 1-4x=0 \Rightarrow x=\frac{1}{4}$ STACIONARNA TAČKA

$$y'=\frac{1-4x}{1+x-2x^2}$$

$$y''=\frac{-4(1+x-2x^2)-(1-4x)\cdot(1-4x)}{(1+x-2x^2)^2}=\frac{-4-\cancel{4x}+8x^2-1+\cancel{4x}+\cancel{4x}-16x^2}{(1+x-2x^2)^2}$$

$$y''=\frac{-8x^2+4x-5}{(1+x-2x^2)^2} \qquad y''\left(\tfrac{1}{4}\right)=\frac{-8\cdot\frac{1}{16}+4\cdot\frac{1}{4}-5}{\left(1+\frac{1}{4}-2\cdot\frac{1}{16}\right)^2}=\frac{-\frac{1}{2}-4}{\left(\frac{5}{4}-\frac{1}{8}\right)^2}<0 \ \max$$

$$\boxed{T_{max}\left(\frac{1}{4},\ln\frac{9}{8}\right)}$$

$$y\left(\tfrac{1}{4}\right)=\ln\left(1+\tfrac{1}{4}-2\cdot\tfrac{1}{16}\right)=\ln\left(\tfrac{5}{4}-\tfrac{1}{8}\right)=\ln\frac{9}{8}$$

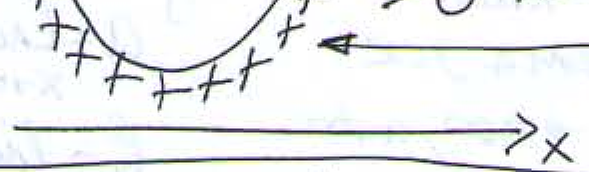
10° Konkavnost $y''>0$ ◡ KRIVINA U LEVO

Konveksnost $y''<0$ ◠ KRIVINA U DESNO

$$y''=\frac{-8x^2+4x-5}{\underbrace{(1+x-2x^2)^2}_{>0\ \forall x\in D_f}} \Rightarrow -8x^2+4x-5>0 /\cdot(-1) \text{ za KONKAVNOST}$$

$$8x^2-4x+5<0$$

$$x_{1/2}=\frac{4\pm\sqrt{16-160}}{16}=\frac{4\pm i\sqrt{144}}{16}=\frac{4\pm 12i}{16}$$



NEMA REALNIH REŠENJA PA $8x^2-4x+5$ NE MOŽE ZA NIJEDNO x DA BUDE <0.

t.j. $-8x^2+4x-5<0 \ \forall x\in D_f$.

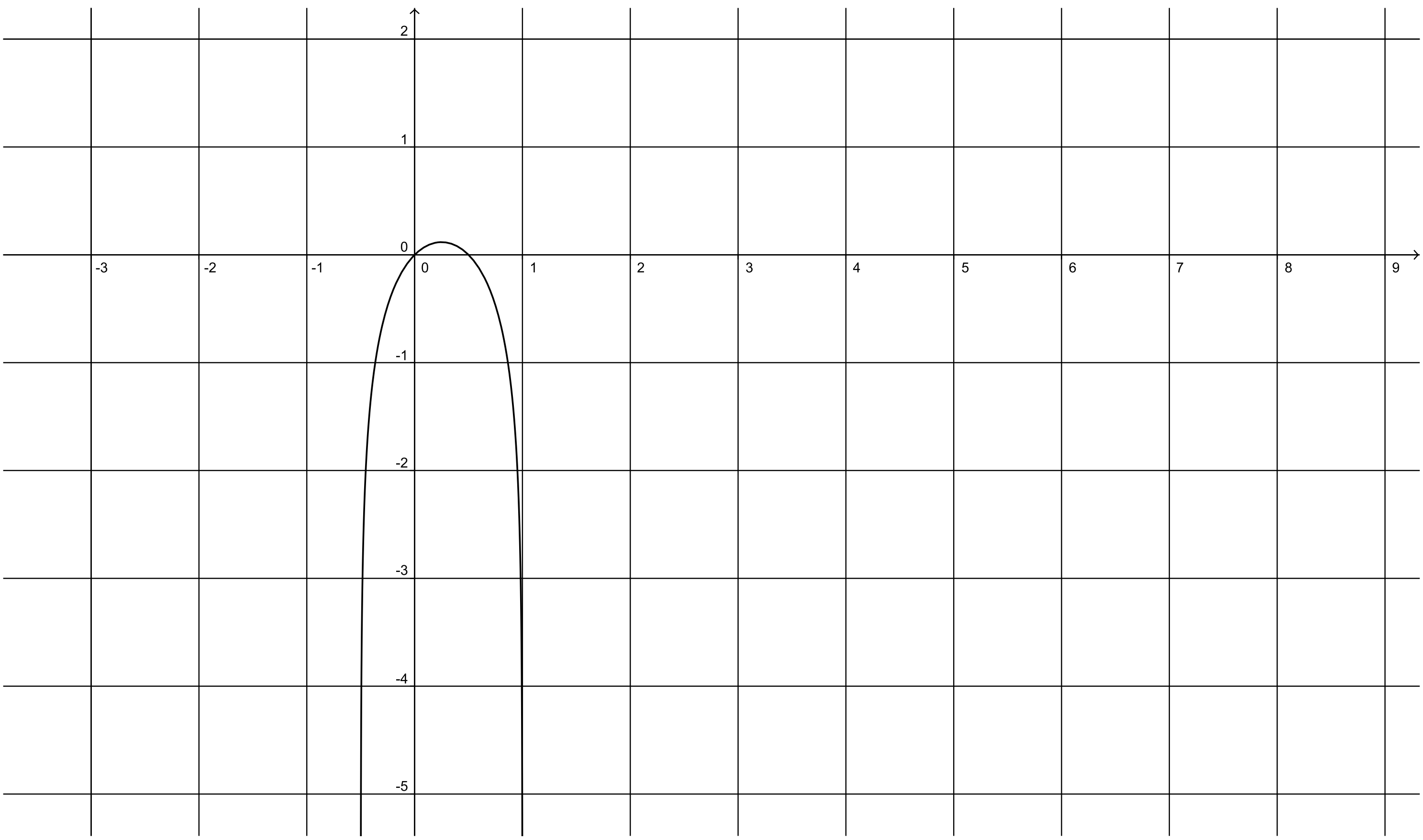
pa $y''=\frac{-}{+}<0 \ \forall x\in D_f$

F-ja u svojoj D_f je uvek KONVEKSNA ◠

11° $y''=0$ PREVOJNE TAČKE

$$-8x^2+4x-5=0$$

$$x_{1/2}=\frac{4\pm 12i}{16} \Rightarrow \text{NEMA PREVOJNIH TAČAKA}$$



2
1
0
-1
-2
-3
-4
-5
-3
-2
-1
0
1
2
3
4
5
6
7
8
9