① $[-\pi, \pi]$ $\quad f(x) = \pi\cdot|x| - x^2$

$f(-x) = \pi\cdot|-x| - (-x)^2 = \pi\cdot|x| - x^2 = f(x)$

$f(-x) = f(x)$ funkcija je PARNA i tada

koeficijenti $\boxed{b_n = 0}$ TRAŽIMO a_0 i a_n!

$$a_0 = \frac{2}{\pi}\int_0^{\pi}\underbrace{\left[\pi\cdot|x| - x^2\right]}_{f(x)}\cdot dx = \frac{2}{\pi}\int_0^{\pi}(\pi x - x^2)dx = \frac{2}{\pi}\cdot\left(\pi\frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_0^{\pi} =$$

$\underline{|x| = x}$ kad $x \in [0, \pi]$

$$= \frac{2}{\pi}\cdot\left(\frac{\pi^3}{2} - \frac{\pi^3}{3} - 0 + 0\right) = \frac{2}{\pi}\cdot\frac{3\pi^3 - 2\pi^3}{6} = \frac{2}{\pi}\cdot\frac{\pi^3}{6} =$$

$$\boxed{a_0 = \frac{1}{3}\pi^2}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}\left[\pi|x| - x^2\right]\cdot\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi}\left[\pi x\cdot\cos nx - x^2\cos nx\right]dx =$$

$$= \frac{2}{\pi}\cdot\pi\underbrace{\int_0^{\pi}x\cdot\cos nx\cdot dx}_{I_1} - \frac{2}{\pi}\underbrace{\int_0^{\pi}x^2\cos nx\,dx}_{I_2} = 2\cdot\left[\frac{x}{n}\cdot\sin nx + \frac{\cos nx}{n^2}\right]\Bigg|_0^{\pi} - \frac{2}{\pi}\cdot\left[\frac{x^2}{n}\cdot\sin nx + \frac{2}{n^2}x\cos nx - \frac{2}{n^3}\sin nx\right]\Bigg|_0^{\pi} =$$

$$= \left(\frac{2}{n^2}\cos n\pi - \frac{2}{n^2}\right) - \frac{2}{\pi}\cdot\left(\frac{2\pi}{n^2}\cdot\cos n\pi - 0\right) = \frac{2}{n^2}\left((-1)^n - 1\right) - \frac{4}{n^2}\cdot(-1)^n = -\frac{2}{n^2}(-1)^n - \frac{2}{n^2} = \boxed{-\frac{2}{n^2}\left[(-1)^n + 1\right] = a_n}$$

$\sin n\pi = 0$
$\sin 0 = 0$
$\cos 0 = 1$
$\cos n\pi = (-1)^n$ VAŽNO!

$\boxed{\int u\cdot dv = u\cdot v - \int v\cdot du}$ PARCIJALNA INTEGRACIJA

$$I_2 = \int x^2\cdot\cos nx\,dx = x^2\cdot\frac{1}{n}\sin nx - \int\frac{1}{n}\cdot\sin nx\cdot 2x\,dx = \frac{x^2}{n}\sin nx - \frac{2}{n}\int x\cdot\sin nx\cdot dx =$$

$u = x^2 \Rightarrow du = 2x\,dx$

$dv = \cos nx\cdot dx \Rightarrow v = \int\cos nx\cdot dx = \frac{1}{n}\cdot\sin nx$

$u = x$, $du = dx$; $dv = \sin nx\,dx$, $v = \int\sin nx\,dx$, $v = -\frac{1}{n}\cos nx$

$$= \frac{x^2}{n}\cdot\sin nx - \frac{2}{n}\cdot\left[x\cdot\left(-\frac{1}{n}\cos nx\right) - \int -\frac{1}{n}\cdot\cos nx\cdot dx\right] = \frac{x^2}{n}\cdot\sin nx + \frac{2}{n^2}x\cos nx - \frac{2}{n^2}\int\cos nx\,dx =$$

$$= \frac{x^2}{n}\cdot\sin nx + \frac{2}{n^2}x\cos nx - \frac{2}{n^3}\cdot\sin nx + C =$$

$$= \boxed{\left(\frac{x^2}{n} - \frac{2}{n^3}\right)\cdot\sin nx + \frac{2}{n^2}\cdot x\cdot\cos nx + C} = I_2$$

$$I_1 = \int x\cdot\cos nx\cdot dx = \frac{x}{n}\cdot\sin nx - \int\frac{1}{n}\cdot\sin nx\cdot dx = \boxed{\frac{x}{n}\cdot\sin nx + \frac{1}{n^2}\cdot\cos nx + C = I_1}$$

$u = x \Rightarrow du = dx$

$dv = \cos nx \Rightarrow v = \int\cos nx\,dx = \frac{1}{n}\cdot\sin nx$

Fourier-ov red bće:

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cdot \cos nx + b_n \cdot \sin nx)$$

$b_n \to 0$

$a_0 = \frac{1}{3}\pi^2$

$a_n = -\frac{2}{n^2}\left[(-1)^n + 1\right] \Rightarrow n = 1, 3, 5, 7, 9, \dots \Rightarrow a_n = 0$

$n = 2, 4, 6, 8, 10, \dots \Rightarrow a_n = -\frac{4}{n^2}$

$b_n = 0$

$$f(x) = \pi \cdot |x| - x^2 = \frac{\pi^2}{6} - 4 \cdot \left(\frac{\cos 2x}{2^2} + \frac{\cos 4x}{4^2} + \frac{\cos 6x}{6^2} + \dots\right) =$$

$$f(x) = \frac{\pi^2}{6} - \frac{4}{2^2} \cdot \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \dots + \dots\right)$$

FOURIER-OV RAZVOJ $f(x) = \pi \cdot |x| - x^2$

za $x = 0 \Rightarrow f(0) = \pi \cdot |0| - 0^2 = 0$

$$0 = \frac{\pi^2}{6} - \left(\frac{\cos 0}{1^2} + \frac{\cos 0}{2^2} + \frac{\cos 0}{3^2} + \dots\right) \Rightarrow$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

ŠTA SMO TREBALI NAĆI