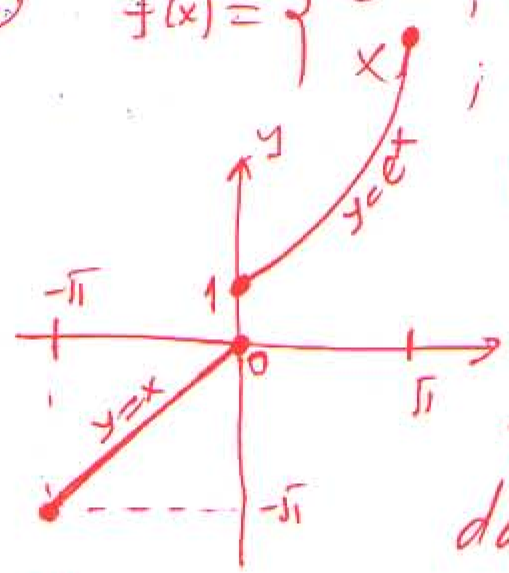
② $f(x)=\begin{cases} e^x & ; x\in[0,\pi] \\ x & ; x\in[-\pi,0] \end{cases}$ $f(x)$ nije parna, nije neparna



U $x=0$ imamo diskontinuitet prve vrste, $y=x$ i $y=e^x$ su monotono rastuće funkcije i obe funkcije imaju konačne vrijednosti na $[-\pi,\pi]$. Time su ispunjeni Dirichlet-ovi uslovi da Fourier-ov red koji ćemo dobiti konvergira.

U tački $0=x$, Fourier-ov red će imati vrijednost:

$$\frac{f(0^+)+f(0^-)}{2}=\frac{e^0+0}{2}=\frac{1}{2}$$

$$a_0=\frac{1}{\pi}\left[\int_{-\pi}^{0}x\,dx+\int_0^{\pi}e^x\,dx\right]=\frac{1}{\pi}\left[\frac{x^2}{2}\Big|_{-\pi}^{0}+e^x\Big|_0^{\pi}\right]=\frac{1}{\pi}\left(-\frac{\pi^2}{2}+e^{\pi}-1\right)=a_0$$

$$a_n=\frac{1}{\pi}\Big[\underbrace{\int_{-\pi}^{0}x\cdot\cos nx\,dx}_{I_1}+\underbrace{\int_0^{\pi}e^x\cdot\cos nx\,dx}_{I_2}\Big]=\frac{1}{\pi}\left[\frac{x}{n}\sin nx+\frac{1}{n^2}\cos nx\Big|_{-\pi}^{0}+\frac{e^x}{1+n^2}(\cos nx+n\sin nx)\Big|_0^{\pi}\right]$$

$$=\frac{1}{\pi}\left[\frac{1}{n^2}-\frac{1}{n^2}(-1)^n\right]+\frac{1}{\pi}\left\{\frac{e^{\pi}}{1+n^2}\left[(-1)^n\right]-\frac{1}{1+n^2}\cdot 1\right\}=\boxed{\frac{1}{n^2\pi}\left[1-(-1)^n\right]+\frac{1}{(1+n^2)\pi}\left[e^{\pi}(-1)^n-1\right]=a_n}$$

$\sin 0=0$
$\sin n\pi=0$
$\cos 0=1$
$\cos n\pi=(-1)^n$

$$I_1=\int x\cdot\cos nx\,dx=\frac{x}{n}\cdot\sin nx-\int\frac{1}{n}\cdot\sin nx\cdot dx=\boxed{\frac{x}{n}\cdot\sin nx+\frac{1}{n^2}\cos nx+C=I_1}$$

$u=x\Rightarrow du=dx$
$dv=\cos nx\Rightarrow v=\int\cos nx\,dx=+\frac{1}{n}\cdot\sin nx$

$$I_2=\int e^x\cdot\cos nx\,dx=e^x\cos nx+n\int e^x\cdot\sin nx\,dx=(*)$$

$u=\cos nx\Rightarrow du=-n\sin nx\,dx$
$dv=e^x dx\Rightarrow v=\int e^x dx=e^x$

$u=\sin nx\Rightarrow du=n\cdot\cos nx\,dx$
$dv=e^x dx\Rightarrow v=e^x$

$$(*)=e^x\cos nx+n\cdot\Big[e^x\cdot\sin nx-n\underbrace{\int e^x\cos nx\,dx}_{I_2}\Big]$$ IMAMO REKURENTNU VEZU

$$I_2=e^x\cos nx+ne^x\cdot\sin nx-n^2 I_2\Rightarrow I_2(1+n^2)=e^x(\cos nx+n\cdot\sin nx)$$

$$\boxed{I_2=\frac{e^x}{1+n^2}\cdot(\cos nx+n\cdot\sin nx)}$$

$$b_n = \frac{1}{\pi}\left[\underbrace{\int_{-\pi}^{0} x\cdot \sin nx\,dx}_{I_3} + \underbrace{\int_0^{\pi} e^x \sin nx\,dx}_{I_4}\right] = \frac{1}{\pi}\left[\frac{\pi}{n}(-1)^n + \frac{n}{1+n^2}\left[1-e^{\pi}\cdot(-1)^n\right]\right]$$

I Konačno: $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$

$$f(x) = \frac{1}{2\pi}\left(-\frac{\pi^2}{2} + e^{\pi} - 1\right) + \sum_{n=1}^{\infty}\left[\frac{1}{n^2\pi}\left[1-(-1)^n\right] + \frac{1}{\pi(1+n^2)}\left[e^{\pi}\cdot(-1)^n - 1\right]\right]\cdot \cos nx + \left[\frac{1}{\pi}\left[\frac{\pi}{n}(-1)^n + \frac{n}{1+n^2}\left[1-e^{\pi}(-1)^n\right]\right]\right]\cdot \sin nx$$

:)

$$I_3 = \int x\cdot \sin nx\,dx = -\frac{x}{n}\cos nx + \frac{1}{n}\int \cos nx\,dx = \boxed{-\frac{x}{n}\cos nx + \frac{1}{n^2}\cdot \sin nx + C}$$

$u = x \Rightarrow du = dx$

$dv = \sin nx\,dx \Rightarrow v = -\frac{1}{n}\cos nx$

$$\int_{-\pi}^{0} x\cdot \sin nx\,dx = -\frac{x}{n}\cos nx + \frac{1}{n^2}\cdot \sin nx\Big|_{-\pi}^{0} = \frac{\pi}{n}(-1)^n$$

$$I_4 = \int e^x\cdot \sin nx\,dx = e^x\cdot \sin nx - \int n\cdot e^x \cos nx\,dx = (*)$$

$u = \sin nx \Rightarrow du = n\cdot \cos nx\cdot dx$

$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$

$u = \cos nx \Rightarrow du = -n\cdot \sin nx\,dx$

$dv = e^x\,dx \Rightarrow v = e^x$

$$(*) = e^x\cdot \sin nx - n\cdot\left[e^x\cdot \cos nx + n\underbrace{\int e^x\cdot \sin nx\,dx}_{I_4}\right]$$ REKURZIJA

$$I_4 = e^x\cdot \sin nx - n e^x \cos nx - n^2\cdot I_4$$

$$I_4(1+n^2) = e^x\cdot(\sin nx - n\cdot \cos nx)$$

$$\boxed{I_4 = \frac{e^x}{1+n^2}(\sin nx - n\cdot \cos nx) + C}$$

$$\int_0^{\pi} e^x \sin nx\,dx = \frac{e^x}{1+n^2}(\sin nx - n\cdot \cos nx)\Big|_0^{\pi} = \frac{-e^{\pi} n}{1+n^2}\cdot(-1)^n + \frac{n}{1+n^2} = \frac{n}{1+n^2}\left[1-e^{\pi}(-1)^n\right]$$