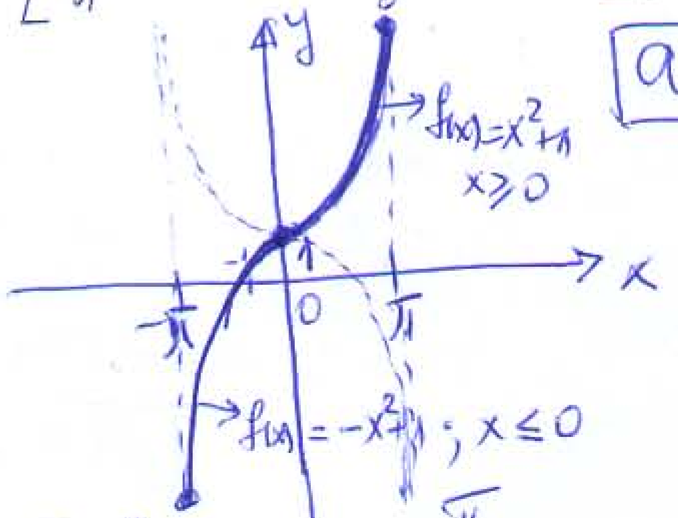
③ $f(x)=x\cdot|x|+1 \quad x\in[-\pi,\pi] \longleftarrow |x|=\begin{cases}x; & x\geq 0\\ -x; & x<0\end{cases}$

$$f(x)=\begin{cases}x^2+1\ ; & x\in[0,\pi] \text{ jer tu je } \cancel{|x|=x} \text{ jer je } x\geq 0\\ -x^2+1\ ; & x\in[-\pi,0] \text{ jer tu je } |x|=-x \text{ jer je } x<0\end{cases}$$

$$a_0=\frac{1}{\pi}\left[\int_{-\pi}^{0}(-x^2+1)dx+\int_{0}^{\pi}(x^2+1)dx\right]=\frac{1}{\pi}\left[\left(-\frac{x^3}{3}+x\right)\Big|_{-\pi}^{0}+\left(\frac{x^3}{3}+x\right)\Big|_{0}^{\pi}\right]=\cancel{\ldots}$$

$$\boxed{a_0=2\cancel{\pi}\cdot\frac{1}{\cancel{\pi}}=2}$$

$= \cancel{\ldots}$



$$\boxed{a_0=2}$$

$$a_n=\frac{1}{\pi}\left[\int_{-\pi}^{0}(-x^2+1)\cos nx\,dx+\int_{0}^{\pi}(x^2+1)\cdot\cos nx\,dx\right]=\frac{1}{\pi}\left[\underbrace{\int_{-\pi}^{0}(-x^2\cos nx)dx}_{I_1}+\int_{-\pi}^{0}\cos nx\,dx+\right.$$

$$\left.+\int_{0}^{\pi}(x^2\cos nx)dx+\int_{0}^{\pi}\cos nx\,dx\right]=\frac{1}{\pi}\left[-\left(\frac{x^2}{n}-\frac{2}{n^3}\right)\sin nx-\frac{2x}{n^2}\cos nx\Big|_{-\pi}^{0}+\frac{1}{n}\sin nx\Big|_{-\pi}^{0}+\right.$$

$$\left.+\left(\frac{x^2}{n}-\frac{2}{n^3}\right)\sin nx+\frac{2x}{n^2}\cos nx\Big|_{0}^{\pi}+\frac{1}{n}\sin nx\Big|_{0}^{\pi}\right]=\frac{1}{\pi}\cdot\left[\cancel{-\frac{2\pi}{n^2}(-1)^n}+\cancel{\frac{2\pi}{n^2}(-1)^n}\right]=\frac{1}{\pi}\cdot 0=\boxed{0=a_n}$$

$$I_1=\int x^2\cos nx\,dx=\frac{x^2}{n}\sin nx-\frac{1}{n}\int 2x\cdot\sin nx\,dx=\frac{x^2}{n}\sin nx-\frac{2}{n}\left[-\frac{x}{n}\cos nx+\frac{1}{n}\int\cos nx\,dx\right]=$$

$u=x^2 \Rightarrow du=2x\,dx$

$dv=\cos nx\,dx \Rightarrow v=\frac{1}{n}\sin nx$

$u=x \Rightarrow du=dx$

$dv=\sin nx\,dx \Rightarrow v=-\frac{1}{n}\cos nx$

$$=\frac{x^2}{n}\sin nx+\frac{2x}{n^2}\cos nx-\frac{2}{n^2}\cdot\frac{1}{n}\sin nx+C=\boxed{\left(\frac{x^2}{n}-\frac{2}{n^3}\right)\sin nx+\frac{2x}{n^2}\cos nx+C=I_1}$$

$$I_2=\int\cos nx\,dx=\boxed{\frac{1}{n}\sin nx+C}$$

$$I_3=\int x^2\sin nx\,dx=-\frac{x^2}{n}\cos nx+\frac{2}{n}\int x\cos nx\,dx=-\frac{x^2}{n}\cos nx+\frac{2}{n}\left[\frac{x}{n}\sin nx-\int\frac{1}{n}\sin nx\,dx\right]=$$

$u=x^2 \Rightarrow du=2x\,dx$

$dv=\sin nx\,dx \Rightarrow v=\int\sin nx\,dx=-\frac{1}{n}\cos nx$

$u=x \Rightarrow du=dx$

$dv=\cos nx\,dx \Rightarrow v=\frac{1}{n}\sin nx$

$$=-\frac{x^2}{n}\cos nx+\frac{2x}{n^2}\sin nx+\frac{2}{n^3}\cos nx+C\Rightarrow\boxed{I_3=\left(\frac{2}{n^3}-\frac{x^2}{n}\right)\cos nx+\frac{2x}{n^2}\cdot\sin nx+C}$$

$$\boxed{I_4=\int\sin nx\,dx=-\frac{1}{n}\cos nx+C}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-x^2+1)\sin nx\,dx + \int_{0}^{\pi}(x^2+1)\sin nx\,dx\right] =$$

$$= \frac{1}{\pi}\left[-\underbrace{\int_{-\pi}^{0} x^2 \sin nx\,dx}_{I_3} + \underbrace{\int_{-\pi}^{0} \sin nx\,dx}_{I_4} + \int_{0}^{\pi} x^2 \sin nx\,dx + \int_{0}^{\pi} \sin nx\,dx\right] =$$

$$= \frac{1}{\pi}\cdot\left[-\left(\frac{2}{n^3}-\frac{x^2}{n}\right)\cos nx - \frac{2x}{n^2}\cancelto{0}{\sin nx}\Big|_{-\pi}^{0} + \left(-\frac{1}{n}\right)\cos nx\Big|_{-\pi}^{0} + \left(\frac{2}{n^3}-\frac{x^2}{n}\right)\cos nx + \frac{2x}{n^2}\cancelto{0}{\sin nx}\Big|_{0}^{\pi} + \left(-\frac{1}{n}\right)\cos nx\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\frac{2}{n^3} + \left(\frac{2}{n^3}-\frac{\pi^2}{n}\right)(-1)^n \cancel{-\frac{1}{n}} + \frac{1}{n}(-1)^n + \left(\frac{2}{n^3}-\frac{\pi^2}{n}\right)(-1)^n - \left(\frac{2}{n^3}-\frac{0^2}{n}\right)\cdot 1 - \frac{1}{n}(-1)^n \cancel{+\frac{1}{n}\cdot 1}\right] =$$

$$= \frac{1}{\pi}\left[-\frac{4}{n^3} + (-1)^n\cdot\left(\frac{4}{n^3}-\frac{2\pi^2}{n}\right)\right] = \frac{1}{\pi}\left[\frac{4}{n^3}\left((-1)^n-1\right) - \frac{(-1)^n}{n}\cdot 2\pi^2\right] =$$

$$\boxed{b_n = \frac{4}{\pi n^3}\left[(-1)^n+1\right] - \frac{(-1)^n}{n}\cdot 2\pi}$$

Konačno:

$$f(x) = 1 + \sum_{n=1}^{\infty}\left[\frac{4\left[(-1)^n+1\right]}{\pi n^3} - \frac{(-1)^n\cdot 2\pi}{n}\right]\sin nx\,dx$$