$x^2+y^2+z^2=9 \Longrightarrow 2(x+y+z)-xyz\leq 10$

$2(x+y+z)-xyz\leq 10 \iff 2(x+y+z)\leq xyz+10$

$\iff 2(x+y+z)\leq 9+1+xyz = x^2+y^2+z^2+xyz+1$

Значи $2(x+y+z)-xyz\leq 10 \iff 2(x+y+z)\leq x^2+y^2+z^2+xyz+1$

Од $AM\geq GM \Rightarrow 2(x+y+z)\geq 2\cdot 3\sqrt[3]{xyz}$.

Ние ќе докажеме дека $6\cdot\sqrt[3]{xyz}\leq x^2+y^2+z^2+xyz+1$ [illegible]

ќе важи и $2(x+y+z)\leq x^2+y^2+z^2+xyz+1$ (што што требаше да докажеме)

$6\sqrt[3]{xyz}\leq x^2+y^2+z^2+xyz+1 \iff 6\sqrt[3]{xyz}\leq 9+1+xyz$

$6\sqrt[3]{xyz}\leq 10+xyz$ $\quad | \; 10+xyz\geq 2\sqrt{10xyz}$ од $AM\geq GM$

Значи ако докажеме дека важи $2\sqrt{10xyz}\geq 6\sqrt[3]{xyz}$ ќе важи и $10+xyz\geq 6\sqrt[3]{xyz}$

$2\sqrt{10xyz}\geq 6\sqrt[3]{xyz} \iff \sqrt{10xyz}\geq 3\sqrt[3]{xyz} \iff (10xyz)^3\geq 3^6(xyz)^2$

$\iff 10^3(xyz)^3\geq 3^6\cdot(xyz)^2 \iff 10^3\cdot xyz\geq 3^6 \iff \frac{10^3}{3^6}\geq\frac{1}{xyz}$

Од $AM\geq GM \Rightarrow 9=x^2+y^2+z^2\geq 3\sqrt[3]{x^2y^2z^2} \iff 3\geq\sqrt[3]{(xyz)^2} \iff 3^3\geq (xyz)^2$

$\iff 3\sqrt{3}\geq xyz$

Бидејќи $3\sqrt{3}\geq xyz \Rightarrow \frac{1}{xyz}\geq\frac{1}{3\sqrt{3}}$

Значи треба да докажеме: $\frac{10^3}{3^6} \geq \frac{1}{3\sqrt{3}}$ –

$10^3 \geq \frac{3^6}{3\sqrt{3}} \iff 10^3 \geq \frac{3^5}{\sqrt{3}} \iff 10^3 \geq 3^4 \cdot \sqrt{3}$

Треба да докажеме $10^3 \geq 3^4 \cdot \sqrt{3}$. $\iff 10^6 \geq 3^9$

$10^6 \geq 3^9 \iff 1000000 \geq 19683$